

# A Novel State Observer For Dynamical Systems with Inaccessible Outputs

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**Abstract**—This paper presents a state observer based on the action reaction law of dynamics. The proposed observer allows estimating states of single input flexible dynamical systems with unknown or inaccessible outputs where the instantaneous system reaction is utilized as a feedback like force/torque and used in the design of a state observer. Necessary and sufficient conditions for observability of this class of dynamical systems are investigated. Robustness of the proposed state observer to parameter uncertainties is further studied. The proposed observer makes it possible to keep a class of single input flexible dynamical systems free from any attached sensors while estimating their states. Validity of the proposed action reaction based state observer is evaluated experimentally.

**Keywords**—Action-reaction state observer, reaction force observer, disturbances, motion control.

## I. INTRODUCTION

Interest in state observers is ever-growing due to the physical flexibility they add to any control system. Technical limitations such as measurements uncertainties, limited bandwidth problems and the complicity of electronic setups associated with each embedded sensor to the system are partially avoided when proper state observers are designed and used. Therefore, dynamical system state estimation is indeed an important task in majority of nowadays motion control applications. The well-known Luenberger observer provides a comprehensive solution for the estimation problem where system states can be observed along with disturbances that can be considered as states providing that dynamical system model is known a priori, input is known and outputs can be measured.

The purpose of the present work is to estimate dynamical system states without taking any measurement from the system side. Outputs of the system are assumed to be unknown or inaccessible. In addition, system parameters are not accurately identified.

At first sight, the previous claims would make it impossible for the estimated states to converge to the actual ones since system outputs are not measured. However, the output measurement is replaced with a natural feedback, namely the incident reaction force/torque on the point/plane of interface between the dynamical system and an attached actuator. The idea is simple and mainly based on realization of the action-reaction law of dynamics through the well-known disturbance observer. It is commonly believed that in order to perform

a motion control assignment, a dynamical system has to be excited by the mean of at least one actuator. Hereafter, the dynamical system instantaneously reacts on the actuator with an equal force/torque to the applied one in the opposite direction. Robustness of a motion control system requires estimation of such reaction force/torque then generating an additional control input to suppress them. Therefore, robustness necessitates two control inputs, the first is a driving input to excite the system, while the second is a compensation input to suppress disturbances. Therefore, one can say with no doubt that in any event a dynamical system will be excited and the incident disturbances have to be realized for sake of robustness. In this work, reaction force/torque is conceptually considered as a natural feedback from the dynamical system on the point/plane of interface between an actuator and a dynamical system. Then this natural feedback is used to design a state observer that does not require taking any measurement from the dynamical system not including the actuator.

Much effort has been expended in the last decades in order to estimate dynamical system states. Reaction force/torque along with actuator force/torque ripple and actuator self-varied mass/inertia are considered as disturbance in [1]-[2]-[4]. Then a disturbance observer is used for the attainment of robust acceleration control by identifying and suppressing the total mechanical load and parameter variation [7]-[18]. The previous observer can be considered as a state observer if disturbances are conceptually defined as system states then used to formulate an augmented state space equation. In this case, the Luenberger observer is more general and can be designed to estimate both system states and disturbances [5]-[6]. The observer is very useful tool for estimating the information of the internal variables of a system that are unknown. However, the main challenge in this application is that the observer is completely dependent on the plant mathematical model accuracy and necessitates measuring the system output that can be inaccessible or unknown. High-gain observer was proposed by Khalil [22] that allows estimating the unmeasured states along with asymptotically attenuating disturbances. Robustness over a range of system uncertainties was enhanced by sliding-mode observer presented by Utkin [23] based on the sliding-mode approach. A non-linear extended state observer was proposed by Han [25] where the non-

linear model is treated as extended state. In addition, the non-linear model along with its derivative are assumed unknown. Thus, achieving inherent robustness as it is independent of the plant mathematical model.

The previous observers differs from each other in the sense of tracking error in transient and steady state, robustness to plant mathematical model and sensitivity to the unknown initial conditions. However, there exist a single feature that they all have in common which is the necessity of measuring system output. But what about if the system output is inaccessible or unknown for some reasons.

With the work of O'Connor [12]-[13]-[14] at which the concept of natural feedback was presented and used to control motion and vibration of non-collocated lumped flexible structures, one can find an answer or at least an idea to the previous question. O'Connor considered the mechanical waves that propagate back and forth between an actuator and end boundary condition as natural feedback from the system which can be used to position a non-collocated point to a target position. However, a measurement from the dynamical system is taken along with ignoring the effect of interaction forces that can adversely affect the performance of the control system. Nevertheless, the natural feedback concept can be used as an alternative for system output to design state observers.

This work is concerned with designing state observers for systems with inaccessible outputs. Based on the action reaction dynamical law, a natural feedback from any dynamical system, namely the incident reaction forces/torques can be obtained from the interface plane of the dynamical system with an attached actuator. Thus, measurements can be focused on the actuator whereas dynamical system can be left free from any measurement whatsoever.

This paper is organized as follow. Problem formulation is presented in Section II where dynamical system is splinted into two portions, actuator and plant side. Output of the plant are inaccessible or unknown therefore actuator is used to estimate the incident reaction force from the plant on the actuator that in turn used to design state observer. State observer is designed in Section III which differs from the well-known Luenberger in the sense of not measuring any of the system outputs. Then, an example is introduced to test the performance of the outlined state observer on a dynamical system with 3 degrees-of-freedom under parameter uncertainties. Experimental results are included in Section IV. Eventually, conclusions and final remarks are included in Section V.

## II. PROBLEM FORMULATION

The dynamical systems we consider can be expressed as

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned}\quad (1)$$

where  $x \in \mathbb{R}^n$  and  $y \in \mathbb{R}^m$  are the state and measurement vectors, respectively.  $A$ ,  $B$ ,  $C$  and  $D$  are the system matrix, distribution vector of input, observation column vector and feed forward matrix with appropriate dimensions, respectively.

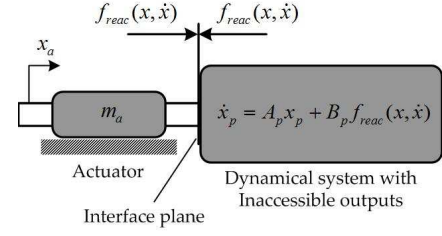


Fig. 1. Reaction force observer.

It can be shown that system (1) can be written for a class of single input multiple outputs flexible system as

$$\dot{x}_a = A_a x_a + B_a u_a + B_{reac} f_{reac}(x, \dot{x}) \quad (2)$$

$$\dot{x}_p = A_p x_p + B_p f_{reac}(x, \dot{x}) \quad (3)$$

where  $x_a$  and  $x_p$  are actuator and plant state vectors, respectively. The subscripts  $(a)$  and  $(p)$  denote the actuator and plant.  $f_{reac}(x, \dot{x})$  is the incident instantaneous reaction force on the actuator,  $B_{reac}$  is the reaction force distribution vector. Figure 1 illustrates the class of dynamical system we consider in this work, the plant states ( $x_p$ ) are inaccessible. This is equivalent to situations at which measurement can not be made or sensor utilization is costly or impractical, e.g., the dynamical system depicted in Fig. 1 can be considered as dynamical system with inaccessible outputs if any of its states ( $x_1, \dots, x_n$ ) can not be measured. The reaction force  $f_{reac}(x, \dot{x})$  is conceptually considered as a feedback-like force which can be used in the design of state observer for the dynamical system (3). Since states of (3) are inaccessible, the feedback-like force  $f_{reac}(x, \dot{x})$  can be estimated from (2) which can be written as follows

$$\dot{x}_a = (A_{an} + \Delta A_a)x_a + (B_{an} + \Delta B_a)u_a + B_{reac} f_{reac}(x, \dot{x}) \quad (4)$$

$$\begin{aligned}A_a &= A_{an} + \Delta A_a \\ B_a &= B_{an} + \Delta B_a\end{aligned}\quad (5)$$

$\Delta A_a$  is a deviation of  $A_a$  and  $\Delta B_a$  is the deviation of  $B_a$  from the nominal values with the subscript  $n$ . Rewriting (3)

$$\dot{x}_a = A_{an}x_a + B_{an}u_a + (\Delta A_a x_a + \Delta B_a u_a + B_{reac} f_{reac}(x, \dot{x})) \quad (6)$$

The third term of (5) is well-known as disturbance ( $d$ ) with force or torque units [2]-[4]

$$d \triangleq \Delta A_a x_a + \Delta B_a u_a + B_{reac} f_{reac}(x, \dot{x}) \quad (7)$$

Applying (6) on the following actuator motion equation

$$(m_a + \Delta m_a)\ddot{x}_a + f_{reac}(x, \dot{x}) = (k_{fn} + \Delta k_f)i_a \quad (8)$$

where  $m_a$ ,  $k_f$  and  $i_a$  are the actuator mass, force constant and current. Disturbance force can be written as

$$d = \Delta m_a \ddot{x}_a + f_{reac}(x, \dot{x}) - \Delta k_f i_a \quad (9)$$

where the first and third terms of the right hand side of (9) are the actuator self-varied mass and torque ripple. In order to

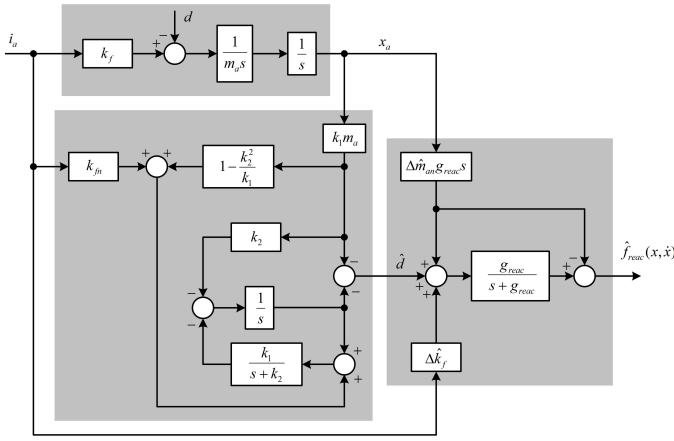


Fig. 2. Reaction force observer.

estimate the feedback-like force  $f_{reac}(x, \dot{x})$ , disturbance force has to be estimated first. Figure 2 illustrates the second order disturbance observer which requires measuring the actuator position along with the its current input. Disturbance observer is then followed by reaction force observer that is depicted in Fig. 2 and can be expressed as

$$\widehat{f_{reac}}(x_a, \dot{x}_a) = \frac{g_{reac}}{s + g_{reac}} [g_{reac} \widehat{\Delta m_a} \dot{x}_a + i_a \widehat{\Delta k_f} + \hat{d}] - g_{reac} \widehat{\Delta m_a} \dot{x}_a \quad (10)$$

$g_{reac}$  is the reaction force observer positive gain.  $k_1$  and  $k_2$  are the disturbance observer gains.  $\widehat{\Delta m_a}$  and  $\widehat{\Delta k_f}$  are the identified actuator parameter deviations. A procedure to determine  $\widehat{\Delta m_a}$  and  $\widehat{\Delta k_f}$  can be found in [29]-[30] through an off-line experiment. The disturbance observer along with the reaction force observer require measuring the actuator position along with the on hand current reference. The incident reaction forces from the dynamical plant are then estimated through these observers and further used in the design of the following action reaction state observer.

### III. ACTION-REACTION STATE OBSERVER

In order to design state observer for the dynamical system (3), the estimated reaction force is conceptually considered as a feedback like force which can be injected onto the structure of a Luenberger like state observer instead of the inaccessible outputs or states. Therefore, the state observer can be written as

$$\dot{\hat{x}} = A\hat{x} + Bu + M(\widehat{f_{reac}}(x_a, \dot{x}_a) - \widehat{f_{reac}}(\hat{x}, \hat{\dot{x}})) \quad (11)$$

the subscript (a) is used to indicate that the estimated reaction force  $\widehat{f_{reac}}(x_a, \dot{x}_a)$  is determined through the actuator variables, whereas,  $\widehat{f_{reac}}(\hat{x}, \hat{\dot{x}})$  is model dependent which depends on states of the overall system (1).  $M$  is the state observer gain vector. Assuming that the actuator is attached to the dynamical system through a flexible element with stiffness  $k$  and an energy dissipating elements with damping  $c$ , ideally, the estimated reaction force can be expressed as follows

$$\widehat{f_{reac}}(x_a, \dot{x}_a) = k(x_a - \hat{x}_1^p) + c(\dot{x}_m - \dot{\hat{x}}_1^p) \quad (12)$$

which is written as explicit function of the actuator states since we need actuator measurements only to estimate the reaction force through (10). On the other hand,  $\widehat{f_{reac}}(\hat{x}, \hat{\dot{x}})$  depends on the estimated states of system (3). Therefore, it can be expressed as

$$\widehat{f_{reac}}(\hat{x}, \hat{\dot{x}}) = k(x_a - \hat{x}_1^p) + c(\dot{x}_a - \dot{\hat{x}}_1^p) \quad (13)$$

subtracting (3) and (11), then after some algebraic manipulations the following estimation error dynamics can be obtained

$$\dot{e} = (I - cML)^{-1}(A + kML)e = \mathcal{A}e \quad (14)$$

$$L = [1 \ 0 \ \dots \ 0]$$

$I \in \mathbb{R}^{n \times n}$  is identity matrix,  $M \in \mathbb{R}^{n \times 1}$ ,  $L \in \mathbb{R}^{1 \times n}$ . (14) indicates that the estimation error will vanish if the matrix  $(I - cML)^{-1}(A + kML)$  is Hurwitz. Therefore, the action reaction state observer vector gain has to be selected such that  $(I - cML)^{-1}(A + kML)$  is Hurwitz which can be achieved through a regular pole placement procedure upon the required behavior of the observer, in general,  $M$  has to be selected such that the observer is at least twice faster than the control system.

### IV. EXPERIMENTAL RESULT

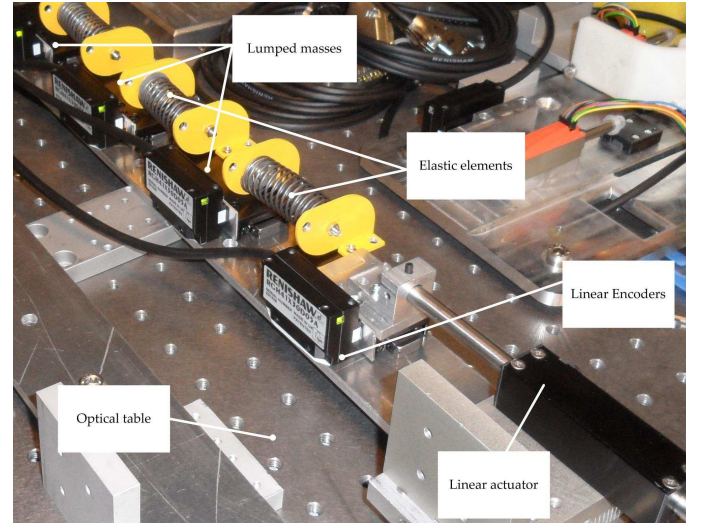
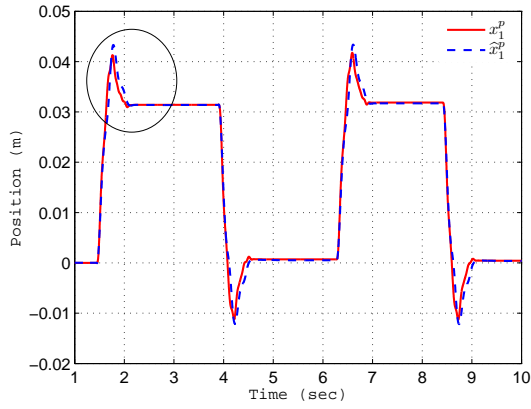
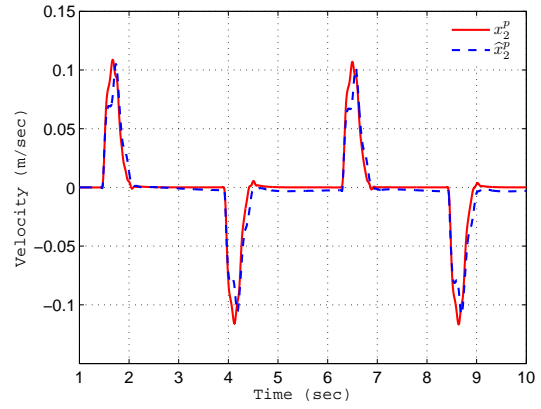


Fig. 3. Experimental setup.

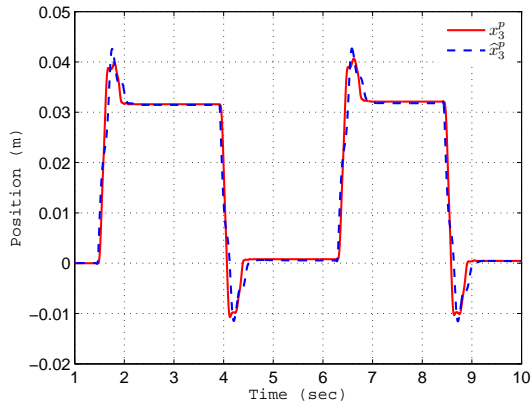
In order to demonstrate the validity of the proposed state observer, experiments are conducted on a single input dynamical system with four degrees of freedom. The experimental setup consists of a linear actuator attached to a flexible lumped mass spring system with three degrees of freedom as shown in Fig. 3. The lumped mass spring system is considered as a plant with inaccessible outputs (3). Therefore, measurements are only allowed to be taken from the actuator side, whereas, plant is kept free from any attached sensors. Experimental parameters are included in Table.I. Actuator position is measured and used as input to the second order disturbance observer depicted in Fig. 2, reaction force is then decoupled out of the



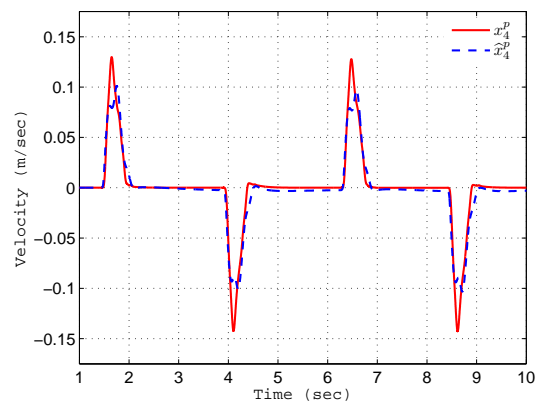
(a)



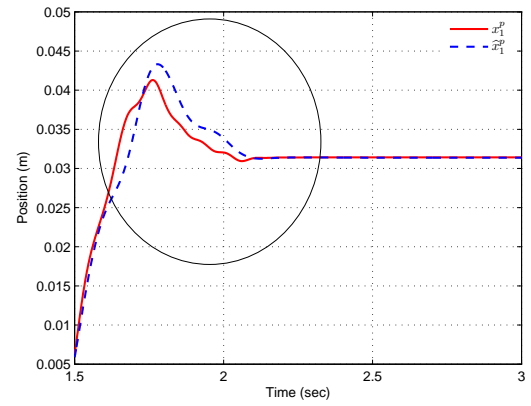
(b)



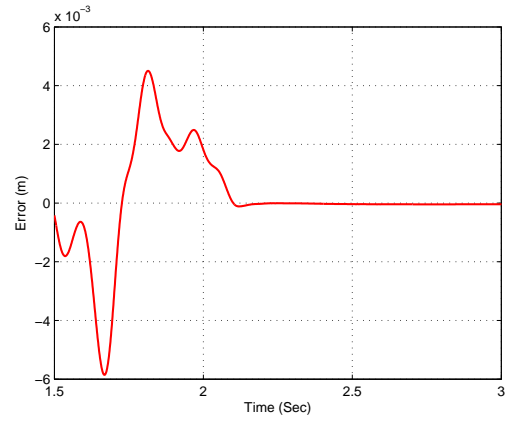
(c)



(d)



(e) Magnified plot



(f) position error

Fig. 4. Experimental states estimation results of a dynamical system with 3-dof ( $x_1^p, x_2^p, x_3^p$  and  $x_4^p$  represent plant first mass position, first mass velocity, second mass position and second mass velocity, respectively).

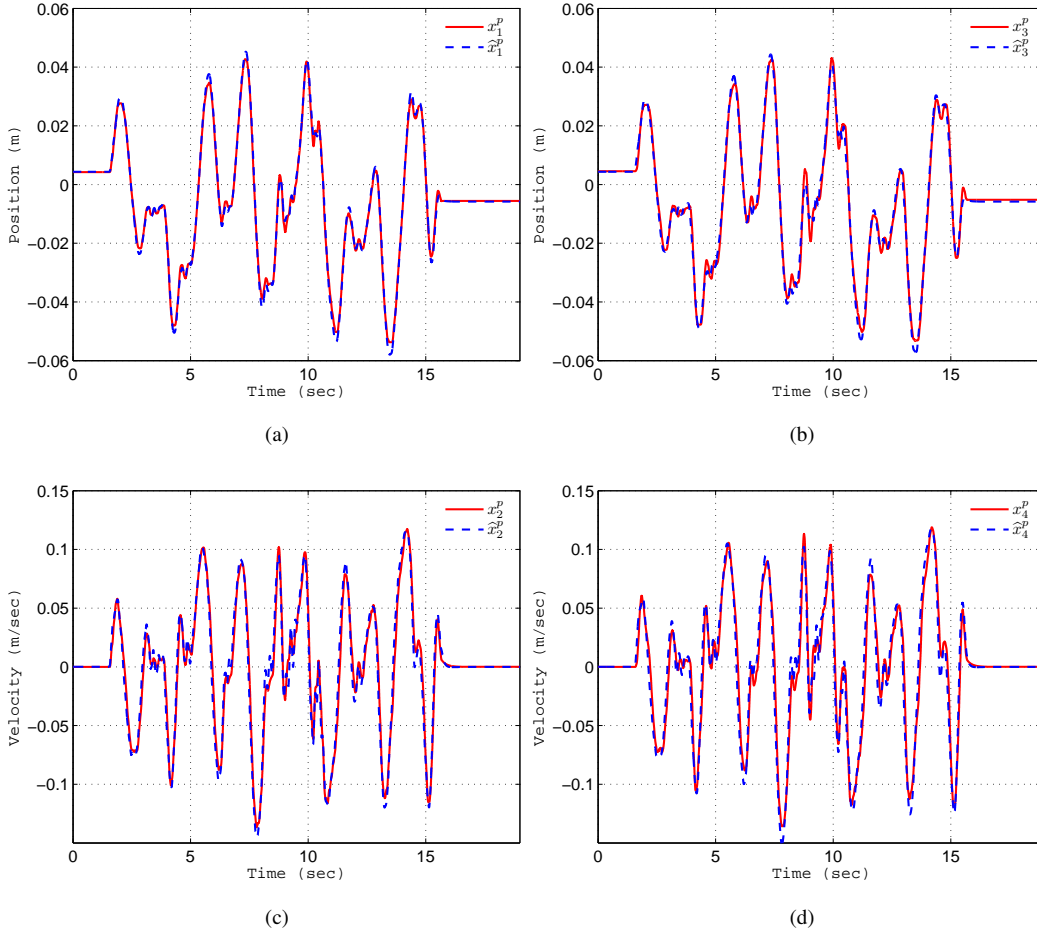


Fig. 5. Experimental states estimation results of a dynamical system with 3-dof ( $x_1^p, x_2^p, x_3^p$  and  $x_4^p$  represent plant first mass position, first mass velocity, second mass position and second mass velocity, respectively).

disturbance force using the reaction force observer (10). The estimated reaction force is then conceptually considered as a feedback-like force which can be used in the design of the state observer (11). Fig. 4 illustrates the difference between the actual and estimated states obtained through (13). In this experiment, the actuator has its own controller which is used to impose arbitrary motions on the dynamical system (4) in order to compare the actual and estimated states. Fig. 3(a) and (b) illustrates the first mass position and velocity along with their estimates. The position tracking error is depicted in Fig. 4(f). Velocity and position of the second mass of the lumped plant along with their estimated are shown in Fig. 4(c) and (d). From the previous experimental results, estimated states converge to the actual ones in approximately less than  $0.2s$ . It is worth noting that this convergence time is not only dependent on the state observer gain vector  $M$ , it also depends on the other second order observer and reaction observer gains  $k_1, k_2$  and  $g_{reac}$ . Fig. 5 illustrates the estimation process when arbitrary motion is imparted to the dynamical system with different amplitudes and frequencies. The difference between the estimated and actual states indicates that the action reaction state observer is satisfactory estimating the dynamical system

states with at most 1.2% error of the peak to peak amplitude. This indicates that the estimated states can be used in the realization of motion control laws of this class of dynamical systems since the state observer can be designed to be at least twice faster than the control system. Experimentally, the second order disturbance gains are adjusted such that  $k_1 = g_{dist}^2$  and  $k_2 = 2g_{dist}$ , where  $g_{dist} = 628 \text{ rad/s}$ . The reaction force observer used throughout the whole experiment was  $g_{reac} = 628 \text{ rad/s}$ . Eventually, the observer gain vector  $M$  can be obtained upon the required performance of the observer. The observer gain vector utilized throughout the whole experiment is

$$M = [0.3 \ 0.1 \ 0.3 \ 0.3 \ 0.1 \ 0.2 \ 3 \ 3]'$$

this yields a negative definite matrix ( $\mathcal{A}$ ) with the following slowest eigenvalues

$$\lambda = -18.51 \pm 1.92i$$

which indicates that the estimated states will exponentially converge to the actual ones according to the error dynamics (14) and can be shown from Fig. 4(f).

## V. CONCLUSION

The problem of designing a state observer for a class of linear dynamical system with single input and multiple inaccessible outputs has been discussed. The incident reaction forces from these dynamical systems can be observed from their actuators then conceptually considered as feedback like forces which can be used in the design of Luenberger like state observer. The main difference between the proposed observe and the Luenberger state observer lays in how the estimation error is generated and injected onto the observer structure. The error signal is generated by subtracting the estimated reaction forces from the estimation based ones rather using the actual states due to the inaccessibility of the plant outputs. The proposed state observer allows keeping a class of dynamical systems with single input and multiple outputs free from any attached sensors while estimating their dynamical states. However, due to the absence of system measurements or outputs, a reduced order state observer can not be realized. The convergence time of the proposed state observer depends on the second order disturbance observer gains ( $k_1$  and  $k_2$ ), reaction force observer gain ( $g_{reac}$ ) and the action reaction state observer gain vector ( $M$ ) which have to be properly selected such that the overall observer is kept twice faster than the control system.

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TABLE I  
EXPERIMENTAL PARAMETERS

Actuator force constant	$k_{fn}$	6.43	N/A
Actuator Nominal mass	$m_{an}$	0.059	kg
Lumped masses	$m_{1,2,3}$	0.019	kg
Force observer gain	$g_{reac}$	628	rad/s
Disturbance observer gain	$g_{dist}$	628	rad/s
Second order observer gain	$k_1$	$g_{dist}^2$	rad/s
Second order observer gain	$k_2$	$2g_{dist}$	rad/s
Sampling time	$T_s$	1	ms

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